

# Born–Infeld-like modified gravity

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A modified theory of gravity with the function  $F(R) = (1 - \sqrt{1 - 2\lambda R})/\lambda$  is suggested and analyzed. At small value of the parameter  $\lambda$  introduced the action is converted into Einstein–Hilbert action. The theory is consistent with local tests which gives a bound on the value of the parameter  $\lambda \leq 2 \times 10^{-6} \text{ cm}^2$ . We have considered the Jordan frame as well as the Einstein frame in which the potential of the scalar field was obtained. The static Schwarzschild–de Sitter solutions of the model are obtained and analyzed. It was demonstrated that the de Sitter space is unstable but a solution with zero Ricci scalar is stable. We show that there is not matter instability in the model suggested.

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## I. INTRODUCTION

The general theory of relativity (GR) based on the Einstein–Hilbert action can not explain the acceleration of the early and late universe. Therefore, GR does not describe precisely gravity and it is reasonable to modify it in such a way that the theory admits the inflation and imitates the dark energy. In addition, the Mach principle is not included in GR. Thus, the economical strategy is not to introduce extra fields to resolve the cosmological problems but to modify the Einstein–Hilbert action by introducing proper  $F(R)$ -gravity. Such theory can describe the inflation and dark energy or, in other words, the early-time inflation and late-time acceleration, and can solve the cosmological constant problem. It should be noted that the modified gravity ( $F(R)$ -gravity) is the phenomenological model describing local tests and observational data. GR was tested in the Solar System in weak-field and slow motion approximation but at the scale of galaxies and clusters and strong gravity has been tested with poor accuracy. The fundamental theory of relativity has to be the renormalizable quantum gravity which may describe quantum of gravitational waves [1]. Corrections introduced by renormalization are quadratic in scalar curvature and are realized in  $R^2$ -gravity but not, in general, in non-linear  $F(R)$ -gravity. There are difficulties to quantize  $F(R)$ -gravity because it is the higher derivative (HD) theory. In HD theories there are additional degrees of freedom and ghosts present so that unitarity of the theory is questionable. Nevertheless,  $F(R)$ -gravity can resolve the coincidence problem, explains the existence of dark matter, and gives detailed account of the transition from deceleration to acceleration [2], [3]. In this paper we do not investigate the cosmological perturbations because within  $F(R)$ -gravity, which is the HD theory, the perturbation theory has not been constructed yet.  $F(R)$ -gravity can be reformulated in a scalar-tensor

form and scalar fields may play the role of dark matter. It should be mentioned that the string theory at low energy gives a scalar-tensor (Brans–Dicke) theory.

The paper is organized as follows. In Sec.2, we formulate a model of modified gravity with the Born–Infeld-like Lagrangian density. We obtain a bound on the parameter  $\lambda$  with the dimension (length)<sup>2</sup>. Static solutions corresponding to de Sitter phase with matter ( $w = -1$ ) and without matter are obtained in Sec.3. In Sec.4, we describe FRW cosmology when the Universe accelerates. The scalar-tensor form of the model is considered and the potential of the scalar field is obtained in Sec.5. We demonstrate that the de Sitter space is unstable and a solution with zero curvature scalar is stable. The cosmological scenario is described. In Sec.6 the matter stability in the model is investigated and we show that there is no matter instability. The summary of results obtained is presented in conclusion.

We use the Minkowski metric of the form  $\eta_{\mu\nu} = \text{diag}(-1, 1, 1, 1)$  and assume that the speed of light  $c$  and Planck’s constant  $\hbar$  to be unity. Greek indices run 0, 1, 2, 3 and Latin indices run the spatial values 1, 2, 3.

## II. THE MODEL OF THE MODIFIED GRAVITATIONAL THEORY

Let us consider  $F(R)$ -gravity with the Lagrangian density

$$\mathcal{L} = \frac{1}{2\kappa^2} F(R) = \frac{1}{2\kappa^2} \frac{1}{\lambda} \left( 1 - \sqrt{1 - 2\lambda R} \right), \quad (1)$$

where  $\kappa = \sqrt{8\pi} M_P^{-1}$ ,  $M_P$  is the Planck mass. Other variants of Born–Infeld type gravity were considered in [4], [5], [6], [7], [8], [9], [10], [11], [12], [13]. It is obvious from equation (1) that the scalar curvature has to obey the restriction  $R \leq 1/(2\lambda)$ . One can say that the smallest size of the universe (during the Big Bang) can not be less than  $\sqrt{2\lambda}$ . Implying that the constant  $\lambda$  with the dimension of (length)<sup>2</sup> is small,  $\lambda R \ll 1$ , we obtain from

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(1) the Taylor series

$$F(R) = R + \frac{1}{2}\lambda R^2 + \frac{1}{2}\lambda^2 R^3 + \dots \quad (2)$$

Thus, at small value of the constant  $\lambda$  introduced, one comes to the Einstein–Hilbert action  $S = 1/(2\kappa^2) \int d^4x \sqrt{-g} R$  ( $g = \det g_{\mu\nu}$ ), as  $\lim_{\lambda \rightarrow 0} F(R) = R$ . Because GR passes local tests the model under consideration also satisfies observational data at the definite bound on  $\lambda$ . The laboratory bound from the Eöt-Wash experiment [14], [15] (see also [16], [17]) gives  $F''(0) \leq 2 \times 10^{-6} \text{ cm}^2$ . From equation (1), we obtain a restriction on the

parameter  $\lambda$ :

$$\lambda \leq 2 \times 10^{-6} \text{ cm}^2. \quad (3)$$

The Taylor series (2) contains all powers in Ricci curvature  $R = g^{\alpha\beta} R_{\alpha\beta}$  ( $R_{\alpha\beta}$  is the Ricci tensor). At small  $\lambda$  series (2) gives the approximate Lagrangian density  $\mathcal{L} = \frac{1}{2\kappa^2} (R + \frac{1}{2}\lambda R^2)$  which was already considered in [18]. Such model, which was motivated to be renormalizable, results in the self-consistent inflation.  $R^2$ -term in this Lagrangian prevents from the singular behavior in the past and in the future [19].

Adding to (1) the Lagrangian of the matter which is the perfect fluid with the energy-momentum tensor  $T_{\alpha\beta}^{(m)} = (p^{(m)} + \rho^{(m)})u_\alpha u_\beta + p^{(m)}g_{\alpha\beta}$  ( $p^{(m)}$  is a pressure,  $\rho^{(m)}$  is the energy density, and the four-velocity of the fluid obeys  $u^\alpha u_\alpha = -1$ ), one obtains equations of motion [2], [3]

$$R_{\mu\nu}F'(R) - \frac{1}{2}g_{\mu\nu}F(R) + g_{\mu\nu}g^{\alpha\beta}\nabla_\alpha\nabla_\beta F'(R) - \nabla_\mu\nabla_\nu F'(R) = \kappa^2 T_{\mu\nu}^{(m)}, \quad (4)$$

where  $\nabla_\mu$  is a covariant derivative,  $F'(R) = dF(R)/dR$ . The conservation of the energy-momentum tensor  $\nabla^\mu T_{\mu\nu}^{(m)} = 0$  gives the equation (for Friedmann–Robertson–Walker (FRW) metric) as follows:

$$\dot{\rho}^{(m)} + 3H(\rho^{(m)} + p^{(m)}) = 0, \quad (5)$$

where  $H = \dot{a}(t)/a(t)$  is the Hubble parameter,  $a(t)$  is a scale factor and a dot above the variable denotes the differentiation with respect to the time. At the particular case when the equation of state (EoS) parameter  $w = p^{(m)}/\rho^{(m)} = -1$ , corresponding to the fluid with the property of the dark energy, the energy density  $\rho^{(m)}$  becomes constant.

### III. STATIC SOLUTIONS

Now, we solve equation (4) for a novel Lagrangian density (1) in a particular case when the Ricci scalar is a constant  $R = R_0$ . We also specify EoS:  $p^{(m)} = -\rho^{(m)}$  with constant  $\rho^{(m)}$ . For constant curvature, equation (4) becomes

$$\frac{1}{2\lambda} \left(1 - \sqrt{1 - 2\lambda R_0}\right) - \frac{R_0}{4\sqrt{1 - 2\lambda R_0}} = \kappa^2 \rho^{(m)}. \quad (6)$$

Solutions to equation (6) are given by

$$R_0 = \frac{1}{2\lambda} - \frac{(2 - b \pm \sqrt{1 - 4b + b^2})^2}{18\lambda}, \quad (7)$$

where  $b = 4\lambda\kappa^2\rho^{(m)}$ . We notice that for our model  $F'(R) = (1 - 2\lambda R)^{-1/2} > 0$  and the regime of antigravity  $F'(R) < 0$  is not realized. In addition, the condition of classical stability for Schwarzschild black holes  $F''(R) \geq 0$  [2], [3] leads to  $F''(R) = \lambda(1 - 2\lambda R)^{-3/2} \geq 0$ , i.e. the constant  $\lambda$  is positive,  $\lambda > 0$  (and  $b > 0$ ). To have the real value of the scalar  $R_0$  in the solution (7), the discriminant should be positive:  $1 - 4b + b^2 \geq 0$ . This inequality possesses two solutions

$$2 - \sqrt{3} \geq b \geq 0, \quad \text{or} \quad b \geq 2 + \sqrt{3}. \quad (8)$$

According to the Solar System test the constant  $\lambda$  has to be small and, therefore, we obtain the bound on the constant  $\lambda$  from the first inequality in (8):

$$0 < \lambda \leq \frac{(2 - \sqrt{3}) M_P^2}{32\pi\rho^{(m)}}. \quad (9)$$

For the region inside the earth  $\rho^{(m)} \simeq 5 \times 10^{18} \text{ eV}^4$ , and using  $M_P \simeq 1.22 \times 10^{28} \text{ eV}$ , we see that the condition (9) can be easily satisfied due to equation (3). Without the matter when  $\rho^{(m)} = 0$ ,  $p^{(m)} = 0$  the parameter  $b = 0$  and solutions (7) read

$$R_0 = 0, \quad \text{or} \quad R_0 = \frac{4}{9\lambda}. \quad (10)$$

It is interesting that at  $b = 4$  from equation (7), one has the same solutions (10) as for free space. For a particular case  $b = 2 - \sqrt{3}$  solutions (7) are degenerated and we arrive at

$$R_0 = \frac{1}{3\lambda}. \quad (11)$$

It follows from (7) that if  $2 - \sqrt{3} > b > 0$ , we have two solutions for  $R_0$  which are positive  $R_0 > 0$ . Thus, both of the solutions lead to the de Sitter space, and we have probably the unification of early-time inflation with late-time acceleration.

Let us consider the spherically symmetric metric with the Schwarzschild form

$$ds^2 = -B(r)dt^2 + \frac{dr^2}{B(r)} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2). \quad (12)$$

Then for EoS  $p^{(m)} = -\rho^{(m)}$  and constant Ricci scalar  $R_0$  all  $F(R)$  theories admit Schwarzschild-(anti)-de Sitter solutions [2], [3] with the function

$$B(r) = 1 - \frac{2MG}{r} - \frac{R_0}{12}r^2, \quad (13)$$

where  $M$  is the mass of the black hole and  $G = \kappa^2/(8\pi)$  is Newton's constant. If  $R_0 > 0$ , we have the de Sitter space and for  $R_0 < 0$ , one has the anti-de Sitter space. According to equation (7) the solutions for the Ricci scalar are positive  $R_0 \geq 0$  and, therefore, in our model for a constant curvature the de Sitter space is realized. For a free space, without the matter ( $p^{(m)} = 0$ ,  $\rho^{(m)} = 0$ ), the non-trivial solution (10)  $R_0 = 4/(9\lambda)$  gives the function (13):

$$B(r) = 1 - \frac{2MG}{r} - \frac{1}{27\lambda}r^2, \quad (14)$$

Thus, the classical stability for Schwarzschild black holes leads again to  $\lambda > 0$ . Comparison of solution (14) with the solution of Einstein's equation with cosmological constant  $\Lambda$  leads to equality  $\Lambda = 1/(9\lambda)$ . This means that

the model suggested mimics the cosmological constant (or the dark energy) for the space without any matter. This is the common property of  $F(R)$  models [20], [21], [2], [3].

The entropy  $S$  in  $F(R)$ -gravity is given by [22], [23], [24], [2], [3]

$$S = \frac{F'(R)A}{4G}, \quad (15)$$

where  $A$  is the area of the horizon. From equation (1), one finds

$$S = \frac{A}{4\sqrt{1 - 2\lambda RG}}. \quad (16)$$

It follows from (16) that instead of Newton constant  $G$ , one can introduce the effective gravitational coupling  $G_{eff} = \sqrt{1 - 2\lambda RG}$ . For the nontrivial solution (10) ( $R_0 = 4/(9\lambda)$ ), we arrive at the effective gravitational constant  $G_{eff} = G/3$ .

#### IV. FRW COSMOLOGY

In homogeneous, isotropic and spatially flat FRW cosmology the space-time metric is given by

$$ds^2 = -dt^2 + a^2(t) (dx^2 + dy^2 + dz^2). \quad (17)$$

The scalar curvature  $R$  is expressed via the the Hubble parameter as follows:  $R = 12H^2 + 6\dot{H}$ . In this case equation (4) reduces to a system of two equations [2]

$$\frac{F(R)}{2} - 3(H^2 + \dot{H})F'(R) + 18(4H^2\dot{H} + H\ddot{H})F''(R) = \kappa^2\rho^{(m)}, \quad (18)$$

$$\frac{F(R)}{2} - (3H^2 + \dot{H})F'(R) + 6(8H^2\dot{H} + 4\dot{H}^2 + 6H\ddot{H} + \ddot{H})F''(R) + 36(4H\dot{H} + \ddot{H})F'''(R) = -\kappa^2 p^{(m)}. \quad (19)$$

For static solutions  $\dot{H}_0 = 0$  and these two equations are consistent with EoS  $\rho^{(m)} = -p^{(m)}$  and reduce to equation (6). Then  $H_0 = \sqrt{R_0/12}$  and from equation (10), one obtains  $H_0 = 1/(3\sqrt{3\lambda})$  for a de Sitter phase. As a result a scale factor becomes

$$a(t) = a_0 \exp\left(\frac{t}{3\sqrt{3\lambda}}\right) \quad (20)$$

and describes the inflation phase.

#### V. THE SCALAR-TENSOR FORM OF THE THEORY

There is a link between modified  $F(R)$ -gravity and scalar-tensor theories of gravity [2], [3]. Equation (1) presents modified gravity in the Jordan frame with metric tensor variables  $g_{\mu\nu}$ . Let us consider the Einstein frame with conformally transformed metric [25]

$$\tilde{g}_{\mu\nu} = F'(R)g_{\mu\nu} = \frac{g_{\mu\nu}}{\sqrt{1 - 2\lambda R}}. \quad (21)$$

Then, in new variables (21), equation (1) becomes

$$\mathcal{L} = \frac{1}{2\kappa^2}\tilde{R} - \frac{1}{2}\tilde{g}^{\mu\nu}\nabla_\mu\varphi\nabla_\nu\varphi - V(\varphi), \quad (22)$$

where  $\tilde{R}$  is defined in new metric (21). We have introduced the scalar field  $\varphi$  and the potential  $V(\varphi)$  as follows:

$$\varphi = -\frac{\sqrt{3}}{\sqrt{2\kappa}} \ln F'(R) = \frac{\sqrt{3}}{\sqrt{2\kappa}} \ln \sqrt{1 - 2\lambda R}, \quad (23)$$

$$V(\varphi) = \frac{RF'(R) - F(R)}{2\kappa^2 F'^2(R)} \Big|_{R=R(\varphi)} = \frac{\phi(1-\phi)^2}{4\lambda\kappa^2}, \quad (24)$$

where  $\phi = \exp(\sqrt{2}\varphi\kappa/\sqrt{3}) = 1/F'(R)$ . The plot of the function (24) is presented in Fig.1. The potential func-

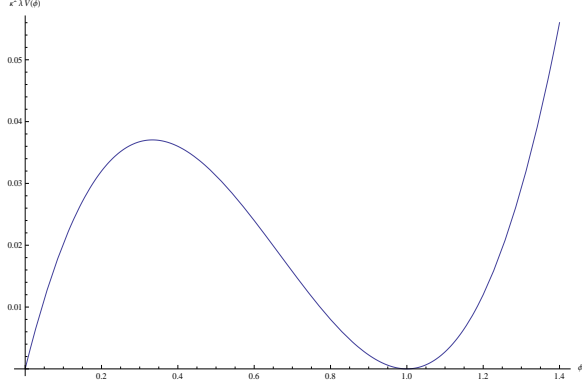


FIG. 1.  $V(\phi)\lambda\kappa^2$  versus  $\phi$ . There is maximum at  $\phi = 1/3$  and minimum at  $\phi = 1$ .

tion (24) possesses minimum at  $\phi = 1$  ( $V''(\phi = 1) > 0$ ) and maximum at  $\phi = 1/3$  ( $V''(\phi = 1/3) < 0$ ). We find that for static solutions (10) the value  $R_0 = 0$  corresponds to the minimum and  $R_0 = 4/(9\lambda)$  corresponds to the maximum of the potential. Thus, the zero scalar curvature is the stable state and the state with Ricci scalar  $R_0 = 4/(9\lambda)$  is unstable. From potential (24), we obtain the mass of a scalar state

$$m_\varphi^2 = \frac{d^2 V}{d\varphi^2} = \frac{\phi(9\phi^2 - 8\phi + 1)}{6\lambda}. \quad (25)$$

It follows from (25) that  $m_\varphi^2 = 1/(3\lambda) > 0$  for  $R_0 = 0$  ( $\phi = 1$ ) and  $m_\varphi^2 = -1/(27\lambda) < 0$  for  $R_0 = 4/(9\lambda)$  ( $\phi = 1/3$ ). Thus, the solution with constant curvature  $R_0 = 4/(9\lambda)$  leads to unstable de Sitter space (12), (14). But the solution with zero scalar curvature  $R_0 = 0$  gives

the condition  $m_\varphi^2 > 0$  for a stability of the Schwarzschild solution (13) ( $R_0 = 0$ ). As the constant  $\lambda$  is small the squared mass  $m_\varphi^2$  is big and corrections to the Newton law are negligible.

One can describe the cosmological scenario as follows. The Universe starts with the large finite positive curvature  $R = 1/(2\lambda)$  (because the parameter  $\lambda$  is small) corresponding to the field  $\phi = 0$ . Then the Universe inflates and has the positive curvature  $R_0 = 4/(9\lambda)$  being in de Sitter's phase. This state is unstable and the Universe rapidly expands with the Hubble parameter  $H_0 = \sqrt{R_0/12}$  (for the constant  $R_0$ ) which defines the expansion rate  $\dot{a}(t)/a(t) = H_0$  of the de Sitter space. Then the scalar curvature decreases (rolls down) and becomes small so that the rate of the expansion  $H$  is small. The final state is stable and corresponds to the vanishing Ricci curvature  $R_0 = 0$ . Thus, the universe approaches stable Minkowski space.

We have described the behavior of the scalar potential of our modified gravity model with the Lagrangian density (1).

## VI. MATTER STABILITY

Now we consider the equation of motion for a curvature scalar. Taking the trace of left and right sides of equation (4), one obtains

$$3g^{\alpha\beta}\nabla_\alpha\nabla_\beta F'(R) + F'(R)R - 2F(R) = \kappa^2 T^{(m)}, \quad (26)$$

where  $T^{(m)} = T_{\mu\nu}^{(m)} g^{\mu\nu}$ . To investigate the matter stability, we follow [26], and apply equation (26) for weak gravity objects. The flat Minkowski metric can be used for weak gravitation so that  $g^{\alpha\beta}\nabla_\alpha\nabla_\beta \simeq \partial_k^2 - \partial_t^2$ . For spatially constant distribution ( $R$  is uniform) equation (26) becomes

$$-3F^{(2)}(R)\ddot{R} - 3F^{(3)}(R)\dot{R}^2 + F^{(1)}(R)R - 2F(R) = \kappa^2 T^{(m)}, \quad (27)$$

where  $F^{(n)}(R) = d^n F(R)/dR^n$ . We consider a perturbative solution  $R = R_0 + R_1$  ( $R_1$  is the perturbed part,  $|R_1| \ll |R_0|$ ), where in the lowest order the curvature, according to GR, is  $R_0 = -\kappa^2 T^{(m)}$  inside the matter and  $R_0 = 0$  outside the matter. From equation (27), one obtains (see [2])

$$\ddot{R}_0 + \ddot{R}_1 + \frac{F^{(3)}(R_0)}{F^{(2)}(R_0)} (\dot{R}_0^2 + 2\dot{R}_0\dot{R}_1) + \frac{2F(R_0) - R_0 [1 + F^{(1)}(R_0)]}{3F^{(2)}(R_0)} = U(R_0)R_1, \quad (28)$$

where

$$U(R_0) = \frac{F^{(3)2} - F^{(2)}F^{(4)}}{F^{(2)2}} \dot{R}_0^2 + \frac{(R_0 F^{(2)} - F^{(1)}) F^{(2)} + (2F - R_0 F^{(1)} - R_0) F^{(3)}}{3F^{(2)2}}. \quad (29)$$

The system is unstable if  $U(R_0) > 0$  because  $R_1$  exponentially increases in the time. We find that for our model

(Eq. (1)) the function  $U(R_0)$  is given by

$$U(R_0) = -\frac{6\lambda^2 \dot{R}_0^2}{(1 - 2\lambda R_0)^2} + \frac{12\lambda R_0 - 7 + 3(2 - \lambda R_0)\sqrt{1 - 2\lambda R_0}}{3\lambda}. \quad (30)$$

One can verify that for static solutions (10)  $U(R_0) < 0$  that indicates on stability of the system. Even for the least value of the scalar curvature  $R_0 = 1/(2\lambda)$ , we have  $U(R_0) < 0$ . As a result, there is no matter instability and the model passes the matter stability test.

## VII. CONCLUSION

Thus, a modified theory of gravity with the Born–Infeld-like action is suggested and analyzed. At limiting case  $\lambda \rightarrow 0$  the action introduced is converted into Einstein–Hilbert action. The Jordan frame as well as the Einstein frame were considered and the poten-

tial of the scalar field in scalar-tensor form of the theory was obtained and is presented in Fig.1. We have found the static Schwarzschild–de Sitter solutions of the model with the de Sitter space to be unstable and the solution with zero Ricci scalar to be stable. The constant  $\lambda$  introduced can be connected with the fundamental length  $l = \sqrt{2\lambda}$  so that the smallest size of the Universe during the Big Bang is  $l$ . From the local tests bound on the constant  $\lambda \leq 2 \times 10^{-6} \text{ cm}^2$ , and we obtain the restriction on the fundamental length  $l \leq 2 \times 10^{-3} \text{ cm}$ . It was demonstrated that no matter instability in the model suggested. There are open questions in the model considered which we leave for further investigations: to obtain exact solutions describing inhomogeneities in a FRW universe, etc.

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